

# SOLUCIONES PARA JUEGOS COOPERATIVOS CON PAGO MÍNIMO

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## 1. ESPACIO DE JUEGOS COMPLETO

**Definition 1.** Dado  $\alpha \in [0, \frac{1}{n}]$ , diremos que  $\varphi$  satisface un  $\alpha$ -pago mínimo si y sólo si

$$(1.1) \quad \varphi_i(v) \geq \alpha v(N)$$

para todo  $v \in G$ .

**Theorem 1.** Para todo  $\alpha \in [0, \frac{1}{n}]$ , existe una única solución

$$\varphi : G \rightarrow \mathbb{R}^N$$

lineal, simétrica, eficiente que satisface  $\alpha$ -pago mínimo. Además está es la solución igualitaria,

$$\varphi_i(v) = \frac{v(N)}{n}.$$

## 2. JUEGOS CONVEXOS

**Definition 2.** Diremos que  $v$  es un juego convexo si y sólo si

$$v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$$

para todo  $S, T \subseteq N$ .

Sea

$$G_c = \{v \in G \mid v \text{ convexo}\}.$$

**Definition 3.** Se dice que  $i$  es un jugador nulo en  $v$  si y sólo si  $v(S \cup \{i\}) = v(S)$  para toda  $S \subseteq N \setminus \{i\}$ .

**Definition 4.** Diremos que  $\varphi$  es cuasi-nula si y sólo si  $\varphi_i(v) = \frac{v(N)}{n}$  para todo jugador nulo en  $v$ .

**Theorem 2.** Para todo  $\alpha \in [0, \frac{1}{n}]$  dado, existe una única solución

$$\varphi : G_c \rightarrow \mathbb{R}^N$$

lineal, simétrica, eficiente y cuasi-nula. Esta es,

$$\varphi_i(v) = (1 - n\alpha)Sh_i(v) + \alpha v(N)$$

para  $i \in N$ . Además está satisface  $\alpha$ -pago mínimo.

In this work we suggest four solutions for cooperative TU games in such a way every player gets at least a minimum payoff, i.e., solutions  $\varphi : G \rightarrow \mathbb{R}^N$  such that for every  $i \in N$  where  $\alpha \in [0, \frac{1}{n}]$  is a given real number.

Three of the this solutions are modifications of the general expression for all linear, symmetric and efficient solutions

$$(2.1) \quad \varphi_i^\beta(v) = \frac{v(N)}{n} + \sum_{\substack{S \ni i \\ S \neq N}} \beta_s \frac{v(S)}{s} - \sum_{S \not\ni i} \beta_s \frac{v(S)}{n-s}$$

given by Ruiz, Valenciano and Zarzuelo (1998).

In the first solution, we modified the first term of (2.1), i.e., we keep the same adjustments than  $\varphi^\beta$  do, but we start from an appropriated initial endowment instead of the egalitarian.

Now, notice that for every pair of solutions  $\varphi^{\beta_1}$  and  $\varphi^{\beta_2}$  of the form (2.1) we have that

$$\eta \varphi^{\beta_1} + (1 - \eta) \varphi^{\beta_2} = \varphi^{\eta \beta_1 + (1 - \eta) \beta_2}$$

Our second solution is the linear convex combination between the egalitarian solution and the Shapley value closest to the Shapley value.

The third solution is an homothecy of the set of imputations with respect to the egalitarian solution, and the last one is a least square solution subject to the restriction (1.1).